OPCION A

Problema 1

a) $A' = \begin{pmatrix} 1 & -2 & -1 & -2 \\ -2 & 0 & -a & 2 \\ 0 & 1 & a & -2 \end{pmatrix} |A| = 2 - 3a = 0 \Rightarrow a = {}^{2}/_{3}$

$$Si\ a \neq {}^{2}/_{3}\ Rango(A) = rango(A') = 3 \Rightarrow SCD$$

$$Si\ a = {}^{2}/_{3} \quad A' = \begin{pmatrix} 1 & -2 & -1 & -2 \\ -2 & 0 & -{}^{2}/_{3} & 2 \\ 0 & 1 & {}^{2}/_{3} & -2 \end{pmatrix} \begin{vmatrix} 1 & -2 & -2 \\ -2 & 0 & 2 \\ 0 & 1 & -2 \end{vmatrix} = 10 \neq 0 \Rightarrow rango(A') = 3$$

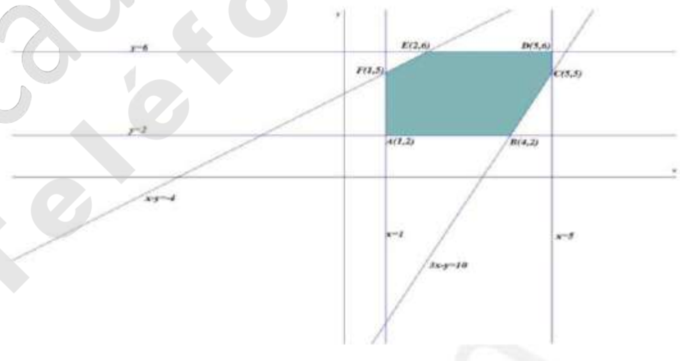
$$rango(A) \neq rango(A') \Rightarrow SI$$

b) $Si\ a = 4 \begin{cases} x - 2y - z = -2 \\ -2x - 4z = 2 \\ y + 4z = -2 \end{cases} \Rightarrow \begin{cases} x = 1 \\ y = 2 \\ z = -1 \end{cases}$

Problema 2

a)

$$Función\ objetivo\ F(x,y) = -200x + 60y S: \begin{cases} 1 \leq x \leq 5 \\ 2 \leq y \leq 6 \\ x - y \geq -4 \\ 3x - y \leq 10 \end{cases} Vértices: (1,2)\ (4,2)\ (5,5)\ (5,6)\ (2,6)\ (1,5)$$



b) $F(1,2) = 1000 \quad F(4,2) = 400\ Mínimo$

$F(5,5) = 2000 \quad F(5,6) = 2600 \quad F(2,6) = 3200\ Máximo \quad F(1,5) = 2800$

Problema 3

a) $Continuidad\ en\ x = -1$

$$\begin{cases} \lim_{x\to -1^-} ax + 1 = a + 1 \\ \lim_{x\to -1^+} x^2 + x - 2 = -2 \end{cases} \Rightarrow -a + 1 = -2 \Rightarrow a = 3$$

b) $Para\ a = 2$

$$f(x) = \begin{cases} 2x + 1\ si\ x < -1 \\ x^2 + x - 2\ si\ x \geqslant -1 \end{cases}$$

$Si\ x < -1\ \ f'(x) = 2\ con\ lo\ que\ crece\ siempre\ en\ (-\infty, -1)\ y\ no\ corta\ a\ los\ ejes$

$Si\ x \geq -1\ f'(x) = 2x + 1 = 0\ \Rightarrow x = {}^{-1}/_{2}\ y\ f''(x) = 2 > 0\ con\ lo\ que\ es\ un\ mínimo.$

$La\ función\ decrece\ en\ (-1, {}^{-1}/_{2})\ y\ crece\ en\ ({}^{-1}/_{2}, \infty)$

$Haciendo\ x = 0\ tendrá\ corte\ con\ OY\ en\ (0, -2)$

$Haciendo\ f(x) = 0\ tendrá\ corte\ con\ OX\ en\ x = 1\ (x = -2\ no\ está\ en\ esta\ rama)$

$luego\ el\ punto\ de\ corte\ es\ (1,0)$

Problema 5

a) $N(\mu, 24) n = 16$

$$P(\overline{X} \geq 48) = 1 - P(\overline{X} \leq 48) 1 - P(Z \leq \frac{18 - 36}{24/\sqrt{16}}) = 1 - P(Z \leq 2 = 1 - 0.9772 = 0.0228$$

b) $E = \frac{47.76 - 24.24}{2} = 11.76$

$$E = Z\alpha/_{2} \frac{\sigma}{\sqrt{n}} = 11.76 = Z\alpha/_{2} \frac{24}{\sqrt{16}} \Rightarrow Z\alpha/_{2} = 1.96$$

$El\ nivel\ de\ confianza\ es\ 95\%$

OPCION B

Problema 1

a) $C=\begin{pmatrix}-1 & 0\\ 3 & 1\end{pmatrix}$ $C^2=\begin{pmatrix}-1 & 0\\ 3 & 1\end{pmatrix}\begin{pmatrix}-1 & 0\\ 3 & 1\end{pmatrix}=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}=I$

$$C^{40}=(C^2)^{20}=I^{20}=I$$

b) $XA+3B=C\Rightarrow\ XA=C-3B\ \Rightarrow X=(C-3B)A^{-1}$

$$C-3B=\begin{pmatrix}-1 & 0\\ 3 & 1\end{pmatrix}-3\begin{pmatrix}1 & 3\\ 2 & -1\end{pmatrix}=\begin{pmatrix}-4 & -9\\ -3 & 4\end{pmatrix}$$

$$A^{-1}=\begin{pmatrix}-1 & -2\\ -1 & -1\end{pmatrix}$$

$$X=\begin{pmatrix}-4 & -9\\ -3 & 4\end{pmatrix}\begin{pmatrix}-1 & -2\\ -1 & -1\end{pmatrix}=\begin{pmatrix}13 & 17\\ -1 & -8\end{pmatrix}$$

Problema 2

a) $Verticales\ en\ x={}^2/_3$

$$\begin{cases}\lim\limits_{x\to {}^2/_3^-}\dfrac{x^2-1}{3x-2}=\left[\dfrac{-5/9}{0^-}\right]=+\infty\\ \lim\limits_{x\to {}^2/_3^+}\dfrac{x^2-1}{3x-2}=\left[\dfrac{-5/9}{0^+}\right]=-\infty\end{cases}$$

$$Horizantales\ \lim_{x\to\pm\infty}\frac{x^2-1}{3x-2}=\pm\infty\ \Rightarrow no\ hay$$

$$Oblicuas\ y=mx+n$$

$$m=\lim_{x\to\pm\infty}\frac{f(x)}{x}=\lim_{x\to\pm\infty}\frac{x^2-1}{3x^2-2}=\frac{1}{3}$$

$$n=\lim_{x\to\pm\infty}f(x)-mx=\lim_{x\to\pm\infty}\frac{x^2-1}{3x-2}-\frac{1}{3}=\lim_{x\to\pm\infty}\frac{2x-3}{3(3x-2)}=\frac{2}{9}$$

$$y=\frac{1}{3}x+\frac{2}{9}$$

b) $f'(x)=\frac{3x^2-4x+3}{(3x-2)^2}=0\Rightarrow 3x^2-4x+3=0\ no\ tiene\ solución$

$$f'(x)>0\ \forall x\in\mathbb{R}-\{{}^2/_3\}\Rightarrow f\ crece\ en\ \mathbb{R}-\{{}^2/_3\}$$

Problema 3

a) $f'(x)=2x+a\ \ f'(2)=4+a=0\Rightarrow a=-4$

$$Como\ f''(x)=2>0\ \Rightarrow x=2\ es\ un\ mínimo$$

b) $Si\ a = -2 \quad f(x) = x^2 - 2x = 0 \ \Rightarrow x = 0\ x = 2$

$$A_1 = \int_0^2 x^2 - 2x\, dx = \left[\frac{x^3}{3} - x^2\right]_0^2 = \frac{8}{3} - 4 = -\frac{4}{3}$$

$$A = |A_1| = \frac{4}{3}$$

Problema 4

a) $P\left({}^{\overline{N}}/_{S}\right) = \frac{P(\overline{N} \cap S)}{P(S)} = \frac{P(S) - P(N \cap S)}{P(S)} = \frac{0.4 - 0.2}{0.4} = 0.5$

b) $P(\overline{N} \cap \overline{S}) = P(\overline{N \cup S}) = 1 - P(N \cup S) = 1 - \left(P(N) + P(S) - P(N \cap S)\right) =$
$= 1 - (0.6 + 0.4 - 0.2) = 0.2$

Problema 5

a) $\sigma = 0.6 \quad n = 100 \quad Z\alpha/_2 = 2.325$

$$E = Z\alpha/_2 \frac{\sigma}{\sqrt{n}} = 2.325 \frac{0.6}{\sqrt{100}} = 0.1395$$

$$IC = (\overline{X} - E, \overline{X} + E) = 6.8605, 7.1395)$$

b)$Z\alpha/_2 = 2.325\ E = 0.1$

$$E = Z\alpha/_2 \frac{\sigma}{\sqrt{n}} = 0.1 = 2.325 \frac{0.6}{\sqrt{n}} \Rightarrow n \geq 194.605 \Rightarrow n = 195$$