OPCION A

Problema 1

a) $|A| = k(k^2 + k - 6) = 0 \Rightarrow k = 0\ k = 2\ k = -3$

$$Si\ k = 0\ k = 2\ y\ k = 3\ \Rightarrow \nexists A^{-1}$$

$$Si\ k \neq 0,2,-3 \Rightarrow \exists A^{-1}$$

b) $X \cdot A = I \Rightarrow X = A^{-1}$

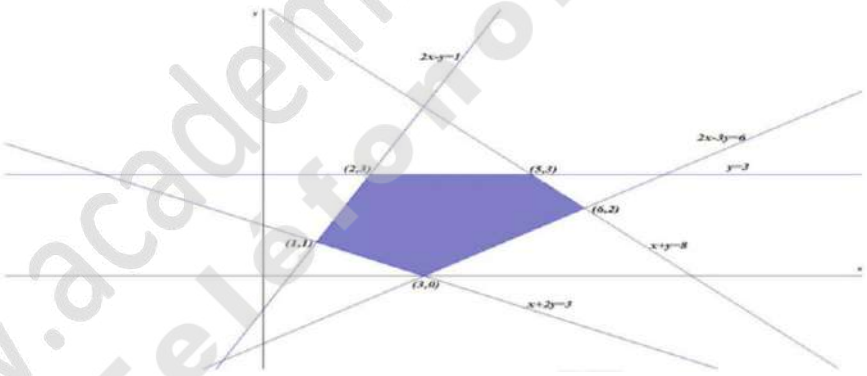
$Si\ k = 1$

$$A = \begin{pmatrix} 1 & -1 & 0 \\ -7 & 1 & 1 \\ -1 & -1 & 1 \end{pmatrix} \Rightarrow A^{-1} = \begin{pmatrix} \frac{-1}{2} & \frac{-1}{4} & \frac{1}{4} \\ \frac{-3}{2} & \frac{-1}{4} & \frac{1}{4} \\ -2 & \frac{-1}{2} & \frac{3}{2} \end{pmatrix}$$

Problema 2

a) $Función\ objetivo\ f(x) = 2x + y$

$$S: \begin{cases} 2x - y > 1 \\ 2x - 3y < 6 \\ x + 2y > 3 \\ x + y < 8 \\ y < 3 \end{cases} \Rightarrow Vértices\ (3,0)\ (2,3)\ (6,2)\ (1,1)\ (5,3)$$



b) $f(3,0) = 6 \quad f(2,3) = 7 \quad f(6,2) = 14\ Máximo$

$f(1,1) = 3\ Mínimo \quad f(5,3) = 13$

Problema 3

a) $Continuidad\ en\ x = 1$

$$\begin{cases} \lim_{x\to 1^-} x^2 + 1 = 2 \\ \lim_{x\to 1^+} \dfrac{ax+b}{x} = a + b \end{cases}$$

$$f(1) = a + b \Rightarrow a + b = 2$$

$Continuidad\ en\ x = 2$

$$\begin{cases} \lim_{x\to 2^-} \dfrac{ax+b}{x} = \dfrac{2a+b}{2} \\ \lim_{x\to 2^+} \sqrt{x^3+1} = 3 \end{cases}$$

$$f(2) = \frac{2a+b}{2} \Rightarrow \frac{2a+b}{2} = 3 \Rightarrow 2a + b = 6$$

$$\begin{cases} a + b = 2 \\ 2a + b = 6 \end{cases} \Rightarrow \begin{cases} a = 4 \\ b = -2 \end{cases}$$

b)

$$S = \int_1^2 \frac{4x-2}{x}\,dx = \int_1^2 (4 - \frac{2}{x})\,dx = [4x - 2\ln|x|]_1^2 = 4 - 2\ln 2 = (4 - ln4)\,u^2$$

Problema 4

a) $P(B/A) = \frac{P(A\cap B)}{P(A)} \Rightarrow P(A \cap B) = \frac{1}{4}\frac{3}{4} = \frac{3}{16}$

$$P(A/B) = \frac{P(A \cap B)}{P(B)} \Rightarrow P(B) = \frac{P(A \cap B)}{P(A/B)} \Rightarrow P(B) = \frac{\frac{3}{16}}{\frac{3}{4}} = \frac{1}{4}$$

$$P(A) \cdot P(B) = \frac{3}{16} = P(A \cap B) \Rightarrow A\ y\ B\ son\ independientes$$

$$P(A \cap B) \neq 0 \Rightarrow A\ y\ B\ son\ incompatibles$$

b) $P(\overline{A}/\overline{B}) = \frac{P(\overline{A}\cap\overline{B})}{P(\overline{B})} = \frac{P(\overline{A\cup B})}{1-P(B)} = \frac{1-(P(A)+P(B)-P(A\cap B))}{1-P(B)} = \frac{1-(\frac{3}{4}+\frac{1}{4}-\frac{3}{16})}{1-\frac{1}{4}} = \frac{1}{4}$

Problema 5

a) $n = 64\ \overline{X} = 30\ \sigma = 5\ Z_{\frac{\alpha}{2}} = 1.96$

$$E = Z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}} = 1.96\frac{5}{\sqrt{64}} = 1.225$$

$$IC = (\overline{X} - E, \overline{X} + E) = (28.775, 31.225)$$

b) $\sigma = 5\ Z_{\frac{\alpha}{2}} = 2.575\ E = 5$

$$E = Z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}} = 5 = 2.575\frac{5}{\sqrt{n}} = 5 \Rightarrow n > 6.63 \Rightarrow n = 7$$

OPCION B

Problema 1

a)$A = \begin{pmatrix} a-1 & 1 & 1 \\ 1 & a-1 & a-1 \\ 1 & 0 & a \end{pmatrix}$

$A' = \begin{pmatrix} a-1 & 1 & 1 & 1 \\ 1 & a-1 & a-1 & 1 \\ 1 & 0 & a & 1 \end{pmatrix} |A| = a^2(a-2) = 0 \Rightarrow a = 0\ a = 2$

$$Si\ a \neq 0\ y\ a \neq 2\ rango(A) = rango(A') = nº\ de\ incognitas \Rightarrow SCD$$

$Si\ a = 0$

$$A = \begin{pmatrix} -1 & 1 & 1 \\ 1 & -1 & -1 \\ 1 & 0 & 0 \end{pmatrix} A' = \begin{pmatrix} -1 & 1 & 1 & 1 \\ 1 & -1 & -1 & 1 \\ 1 & 0 & 0 & 1 \end{pmatrix}$$

$$|A| = 0 \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} \neq 0 \Rightarrow RA = 2$$

$$\begin{vmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 0 & 1 \end{vmatrix} = 2 \neq 0 \Rightarrow rango(A') = 3$$

$$Rango(A) = 2 \neq Rango(A') = 3 \Rightarrow SI$$

$Si\ a = 2$

$A' = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 0 & 2 & 1 \end{pmatrix}$ $Todos\ los\ menores\ de\ orden\ 3\ son\ 0 \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} = -2 \neq 0 \Rightarrow rango(A') = 2$

$$Rango(A) = 2 = rango(A') < nº\ de\ incognitas \Rightarrow SCI$$

b)

$$Si\ a = 3 \begin{cases} 2x + y + z = 1 \\ x + 2y + 2z = 1 \Rightarrow \\ x + 3z = 1 \end{cases}$$

$$x = \frac{\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 0 & 3 \end{vmatrix}}{|A|} = \frac{3}{9} = \frac{1}{3}$$

$$y=\frac{\begin{vmatrix}2 & 1 & 1\\ 1 & 1 & 2\\ 1 & 1 & 3\end{vmatrix}}{|A|}=\frac{1}{9}$$

$$z=\frac{\begin{vmatrix}2 & 1 & 1\\ 1 & 2 & 1\\ 1 & 0 & 1\end{vmatrix}}{|A|}=\frac{2}{9}$$

$$\begin{cases}x=\frac{1}{3}\\ y=\frac{1}{9}\\ z=\frac{2}{9}\end{cases}$$

Problema 2

a) $Continuidad\ en\ x=0\begin{cases}\lim\limits_{x\to 0^-} x^2+2x=0\\ \lim\limits_{x\to 0^+} -x^2+3x=0\end{cases}\Rightarrow la\ función\ es\ continua\ en\ x=0$

$$f(0)=0$$

$$Derivabilidad\ en\ x=0$$

$$f'(x)=\begin{cases}2x+2\ si\ x<0\\ -2x+3\ si\ x\geq 0\end{cases}\Rightarrow\begin{cases}f'(0^-)=2\\ f'(0^+)=3\end{cases}\Rightarrow la\ función\ no\ es\ derivable\ en\ x=0$$

b)

$$x=a\quad m=f'(a)$$

$$si\ x<0\begin{cases}f'(a)=2a+2=-2\ \ a=-2\\ b=f(-2)=0\end{cases}\Rightarrow y=-2(x+2)$$

$$si\ x\geqslant 0\begin{cases}f'(a)=-2a+3=-2\ \ a=\frac{5}{2}\\ b=f(\frac{5}{2})=\frac{5}{4}\end{cases}\Rightarrow y-\frac{5}{4}=-2(x-\frac{5}{2})$$

Problema 3

a) *Asíntotas verticales*

$x=3$

$$\begin{cases}\lim\limits_{x\to 3^-}\frac{x^2-3}{x^2-9}=\left[\frac{6}{0^-}\right]=-\infty\\ \lim\limits_{x\to 3^+}\frac{x^2-3}{x^2-9}=\left[\frac{6}{0^+}\right]=+\infty\end{cases}$$

$x=-3$

$$\begin{cases} \lim_{x\to -3^-} \dfrac{x^2-3}{x^2-9} = \left[\dfrac{6}{0^+}\right] = +\infty \\ \lim_{x\to -3^+} \dfrac{x^2-3}{x^2-9} = \left[\dfrac{6}{0^-}\right] = -\infty \end{cases}$$

Horizontales

$$\lim_{x\to\infty} \frac{x^2-3}{x^2-9} = 1 \Rightarrow y = 1$$

Oblicuas no hay por haber asíntota horizontal.

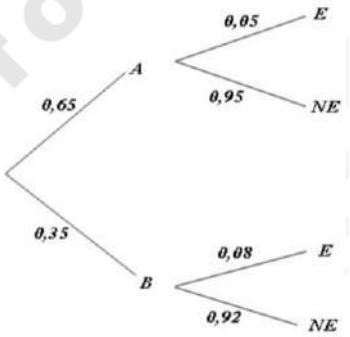
b) $f'(x) = \frac{-12x}{(x^2-9)^2} = 0 \Rightarrow x = 0$

$(-\infty, -3)U(-3,0)$f´(x)<0 Decrece

$(0,3)U(3,\infty)$f´(x)>0 crece

$\Rightarrow Máximo(0, {}^1/_3)$

Problema 4



a) $P(E) = 0.65 \cdot 0.05 + 0.35 \cdot 0.08 = 0.0605$

b) $P({}^A/_E) = \frac{P({}^E/_A)P(A)}{P(E)} = \frac{0.05 \cdot 0.65}{0.0605} = 0.5372$

Problema 5

a) $\sigma = 9 \;\; n = 100 \; Z_{\alpha/2} = 1.645 \;\; \overline{X} = 8.1$

$$E = Z_{\alpha/2}\frac{\sigma}{\sqrt{n}} = 1.645\frac{9}{\sqrt{100}} = 1.4805$$

$$IC = \left(\overline{X} - E, \overline{X} + E\right) = (6.6195, 9.5805)$$

b) $n = 144 \;\; IC = (7.766, 10.233)$

$$\begin{cases} 7.766 = \overline{X} - E \\ 10.233 = \overline{X} + E \end{cases} \Rightarrow \overline{X} = 8.9995 \;\; E = 1.2335$$

$$E = Z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \Rightarrow Z_{\alpha/2} = \frac{1.2335 \cdot \sqrt{144}}{9} = 1.645$$

El nivel de confianza es 90%