

OPCION A

Problema 1

a) $|A \cdot C \cdot C^t \cdot A^{-1}| = |A||C||C^t||A^{-1}| = |A^2| = 4(|C| = |C^t||A| \cdot |A^{-1}| = 1)$

b)

$$M = A \cdot B = \begin{pmatrix} 3 & 2 & 2 \\ 1 & 7 & 4 \\ 4 & 5 & 2 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 5 & 3 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 16 & 11 \\ 37 & 26 \\ 33 & 21 \end{pmatrix}$$

No tiene inversa pues no es cuadrada

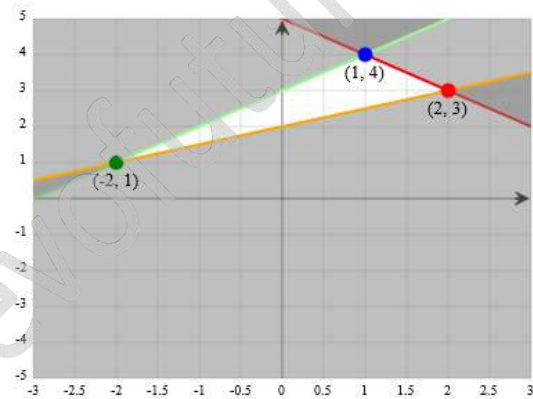
Problema 2

a) Funcion objetivo $f(x,y)=2x+y$

recta verde: $y=x+3$

recta roja: $y=-x+5$

recta amarilla $y=1/2x+2$



Vértice	Rectas tras vértice
● (1, 4)	$x + y = 5$ $x - y = -3$
● (2, 3)	$x + y = 5$ $x - 2y = -4$
● (-2, 1)	$x - y = -3$ $x - 2y = -4$

b) $z=f(-2,1)=-3$ Mínimo

$z=f(1,4)=6$

$z=f(2,3)=7$ Máximo

Problema 3

a) $x^3 + 8 = 0 \Rightarrow x = -2$ Luego hay dos áreas S_1 de $[-3, -2]$ y S_2 en $[-2, -1]$

$$S_1 = \int_{-3}^{-2} (x^3 + 8) dx = \left[\frac{x^4}{4} + 8x \right]_{-3}^{-2} = -\frac{33}{4}$$

$$S_2 = \int_{-2}^{-1} (x^3 + 8) dx = \left[\frac{x^4}{4} + 8x \right]_{-2}^{-1} = \frac{17}{4}$$

$$S = |S_1| + |S_2| = \frac{25}{4}$$

b) $b = f(1) = 9$ $f'(x) = 3x^2$ $m = f'(1) = 3$

$$y - 9 = 3(x - 1)$$

Problema 4

$$P(H) = 0.55 \quad P(M) = 0.45 \quad P(C) = 0.3P(C/M) = 0.25$$

a) $P(M/C) = \frac{P(M) \cdot P(C/M)}{P(C)} = \frac{0.45 \cdot 0.25}{0.30} = 0.375$

b) $P(H \cap C) + P(M \cap C) = P(C)$

$$P(H \cap C) = P(C) - P(M \cap C) = P(C) - P(C/M) \cdot P(M) = 0.3 - 0.25 \cdot 0.45 = 0.1875$$

Problema 5

a) $Z_{\frac{\alpha}{2}} = 1.96 \quad E = 5$

$$E = Z_{\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}} = 1.96 \frac{50}{\sqrt{n}} = 5 \Rightarrow n > 96.04 \Rightarrow n = 97$$

b) $n = 25 \quad \mu = 950 \quad \bar{X} \sim N\left(950, \frac{50}{\sqrt{25}}\right) = N(950, 10)$

$$P(\bar{X} \leq 940) = P\left(Z \leq \frac{940 - 950}{10}\right) = P(Z \leq -1) = 1 - P(Z \leq 1) = 1 - 0.8413 = 0.1587$$

OPCION B

Problema 1

a) $A' = \begin{pmatrix} 1 & 2 & 1 & 1 \\ 1 & 2 & 3 & 0 \\ 1 & a & 2 & 0 \end{pmatrix} \quad |A| = -2a + 4 = 0 \Rightarrow a = 2$

Si $a \neq 2 \quad |A| \neq 0 \Rightarrow \text{rango}(A) = 3 = \text{rango}(A') = 3 = n^\circ \text{ de incógnitas} \Rightarrow \text{SCD}$

Si $a = 2 \quad A' = \begin{pmatrix} 1 & 2 & 1 & 1 \\ 1 & 2 & 3 & 0 \\ 1 & 2 & 2 & 0 \end{pmatrix}$ Todos los menores de orden 3 son 0 $\begin{vmatrix} 2 & 1 \\ 2 & 3 \end{vmatrix} = 4 \neq 0$
 $\Rightarrow \text{rango}(A') = 2 = \text{rango}(A) < n^\circ \text{ de incógnitas} \Rightarrow \text{SCI}$

b) Si $a = 0 \quad \begin{cases} x + 2y + z = 1 \\ x + 2y + 3z = 0 \\ x + 2z = 0 \end{cases} \Rightarrow \begin{cases} x = 1 \\ y = 1/4 \\ z = -1/2 \end{cases}$

Problema 2

a) Continuidad en $x = -1$

$$\lim_{x \rightarrow -1^-} \frac{-x + b}{x - 2} = -\frac{1 + b}{3}$$

$$\lim_{x \rightarrow -1^+} \frac{x^2 + 6x + 5}{x^2 + 4x + 3} = \left[\frac{0}{0} \right] = \text{L'HOPITAL} = \lim_{x \rightarrow -1^+} \frac{2x + 6}{2x + 4} = 2$$

$$-\frac{1+b}{3} = 2 \Rightarrow b = -7$$

b) *Asíntotas*

Verticales no hay porque $x = 2$ no está en la rama $x < -1$

Horizontales

$$\lim_{x \rightarrow -\infty} \frac{-x+b}{x-2} = -1 \Rightarrow y = -1$$

$$\lim_{x \rightarrow \infty} \frac{x^2+6x+5}{x^2+4x+3} = 1 \Rightarrow y = 1$$

Problema 3

a)

$$f(x) = \int 6x^2 + 4x - 3dx = 2x^3 + 2x^2 - 2x + C$$

$$f(0) = 5 \Rightarrow C = 5 \Rightarrow f(x) = 2x^3 + 2x^2 - 2x + 5$$

b) $f'(x) = 6x^2 + 4x - 3 = 0 \Rightarrow x = -1 \quad x = 1/3$

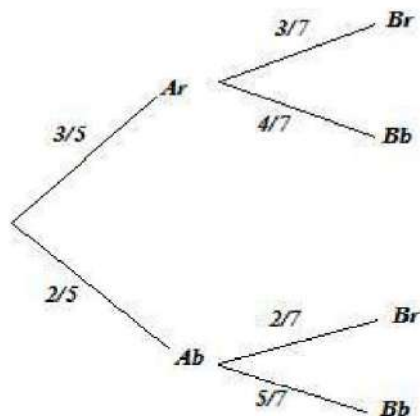
	$(-\infty, -1)$	$(-1, 1/3)$	$(1/3, \infty)$
<i>Signo de $f'(x)$</i>	+	-	+
<i>$f(x)$</i>	<i>Creciente</i>	<i>Decreciente</i>	<i>Creciente</i>

Tiene un mínimo en $(1/3, 125/27)$ y un máximo en $(-1, 7)$

Problema 4

a) $P(B_r) = \frac{3}{5} \cdot \frac{3}{7} + \frac{2}{5} \cdot \frac{2}{7} = \frac{13}{35} = 0.371$

b) $P(A_b \cap B_b) = \frac{2}{5} \cdot \frac{5}{7} = \frac{2}{7} = 0.286$





Problema 5

a) $N(\mu, 650) \sigma = 5 \quad n = 25 \quad Z_{\frac{\alpha}{2}} = 1.96 \quad \bar{X} = 70$

$$E = Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} = 1.96 \frac{5}{\sqrt{25}} = 1.96$$

$$IC = (\bar{X} - E, \bar{X} + E) = (68.04, 71.96)$$

b) $\mu = 70 \quad n = 12 \quad \frac{855}{12} = 71.25$

$$\begin{aligned} P(\bar{X} \geq 71.25) &= 1 - P(\bar{X} \leq 71.25) = 1 - P\left(Z \leq \frac{71.25 - 70}{5/\sqrt{12}}\right) = \\ &= 1 - P(Z \leq 0.87) = 1 - 0.8023 = 0.1977 \end{aligned}$$

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